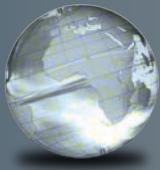


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# Econometric Analysis

EIGHTH EDITION

William H. Greene



EIGHTH EDITION  
GLOBAL EDITION  
ECONOMETRIC ANALYSIS



**William H. Greene**

*The Stern School of Business*

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## BRIEF CONTENTS



### Examples and Applications

### Preface

## **Part I      The Linear Regression Model**

Chapter 1	Econometrics	41
Chapter 2	The Linear Regression Model	52
Chapter 3	Least Squares Regression	68
Chapter 4	Estimating the Regression Model by Least Squares	94
Chapter 5	Hypothesis Tests and Model Selection	153
Chapter 6	Functional Form, Difference in Differences, and Structural Change	193
Chapter 7	Nonlinear, Semiparametric, and Nonparametric Regression Models	242
Chapter 8	Endogeneity and Instrumental Variable Estimation	282

## **Part II      Generalized Regression Model and Equation Systems**

Chapter 9	The Generalized Regression Model and Heteroscedasticity	337
Chapter 10	Systems of Regression Equations	366
Chapter 11	Models for Panel Data	413

## **Part III      Estimation Methodology**

Chapter 12	Estimation Frameworks in Econometrics	505
Chapter 13	Minimum Distance Estimation and the Generalized Method of Moments	528
Chapter 14	Maximum Likelihood Estimation	577
Chapter 15	Simulation-Based Estimation and Inference and Random Parameter Models	681
Chapter 16	Bayesian Estimation and Inference	734

## **Part IV      Cross Sections, Panel Data, and Microeometrics**

Chapter 17	Binary Outcomes and Discrete Choices	765
------------	--------------------------------------	-----

Chapter 18	Multinomial Choices and Event Counts	866
Chapter 19	Limited Dependent Variables—Truncation, Censoring, and Sample Selection	958

**Part V Time Series and Macroeconometrics**

Chapter 20	Serial Correlation	1021
Chapter 21	Nonstationary Data	1062
References		1094

Index		1138
-------	--	------

**Part VI Online Appendices**

Appendix A	Matrix Algebra	A-1
Appendix B	Probability and Distribution Theory	B-1
Appendix C	Estimation and Inference	C-1
Appendix D	Large-Sample Distribution Theory	D-1
Appendix E	Computation and Optimization	E-1
Appendix F	Data Sets Used in Applications	F-1

## Contents



**Examples and Applications**      **25**

**Preface**      **35**

## **Part I The Linear Regression Model**

### **CHAPTER 1 Econometrics**      **41**

1.1	Introduction	41
1.2	The Paradigm of Econometrics	41
1.3	The Practice of Econometrics	43
1.4	Microeconometrics and Macroeconometrics	44
1.5	Econometric Modeling	45
1.6	Plan of the Book	48
1.7	Preliminaries	49
1.7.1	<i>Numerical Examples</i>	49
1.7.2	<i>Software and Replication</i>	50
1.7.3	<i>Notational Conventions</i>	50

### **CHAPTER 2 The Linear Regression Model**      **52**

2.1	Introduction	52
2.2	The Linear Regression Model	53
2.3	Assumptions of the Linear Regression Model	56
2.3.1	<i>Linearity of the Regression Model</i>	57
2.3.2	<i>Full Rank</i>	60
2.3.3	<i>Regression</i>	62
2.3.4	<i>Homoscedastic and Nonautocorrelated Disturbances</i>	63
2.3.5	<i>Data Generating Process for the Regressors</i>	65
2.3.6	<i>Normality</i>	65
2.3.7	<i>Independence and Exogeneity</i>	66
2.4	Summary and Conclusions	67

### **CHAPTER 3 Least Squares Regression**      **68**

3.1	Introduction	68
3.2	Least Squares Regression	68

3.2.1	<i>The Least Squares Coefficient Vector</i>	69
3.2.2	<i>Application: An Investment Equation</i>	70
3.2.3	<i>Algebraic Aspects of the Least Squares Solution</i>	73
3.2.4	<i>Projection</i>	73
3.3	Partitioned Regression and Partial Regression	75
3.4	Partial Regression and Partial Correlation Coefficients	78
3.5	Goodness of Fit and the Analysis of Variance	81
3.5.1	<i>The Adjusted R-Squared and a Measure of Fit</i>	84
3.5.2	<i>R-Squared and the Constant Term in the Model</i>	87
3.5.3	<i>Comparing Models</i>	88
3.6	Linearly Transformed Regression	88
3.7	Summary and Conclusions	89
<b>CHAPTER 4 Estimating the Regression Model by Least Squares</b>		<b>94</b>
4.1	Introduction	94
4.2	Motivating Least Squares	95
4.2.1	<i>Population Orthogonality Conditions</i>	95
4.2.2	<i>Minimum Mean Squared Error Predictor</i>	96
4.2.3	<i>Minimum Variance Linear Unbiased Estimation</i>	97
4.3	Statistical Properties of the Least Squares Estimator	97
4.3.1	<i>Unbiased Estimation</i>	99
4.3.2	<i>Omitted Variable Bias</i>	99
4.3.3	<i>Inclusion of Irrelevant Variables</i>	101
4.3.4	<i>Variance of the Least Squares Estimator</i>	101
4.3.5	<i>The Gauss–Markov Theorem</i>	102
4.3.6	<i>The Normality Assumption</i>	103
4.4	Asymptotic Properties of the Least Squares Estimator	103
4.4.1	<i>Consistency of the Least Squares Estimator of <math>\beta</math></i>	103
4.4.2	<i>The Estimator of Asy. Var[<math>b</math>]</i>	105
4.4.3	<i>Asymptotic Normality of the Least Squares Estimator</i>	106
4.4.4	<i>Asymptotic Efficiency</i>	107
4.4.5	<i>Linear Projections</i>	110
4.5	Robust Estimation and Inference	113
4.5.1	<i>Consistency of the Least Squares Estimator</i>	114
4.5.2	<i>A Heteroscedasticity Robust Covariance Matrix for Least Squares</i>	114
4.5.3	<i>Robustness to Clustering</i>	115
4.5.4	<i>Bootstrapped Standard Errors with Clustered Data</i>	117
4.6	Asymptotic Distribution of a Function of $\mathbf{b}$ : The Delta Method	118
4.7	Interval Estimation	121
4.7.1	<i>Forming a Confidence Interval for a Coefficient</i>	121
4.7.2	<i>Confidence Interval for a Linear Combination of Coefficients: the Oaxaca Decomposition</i>	123

4.8	Prediction and Forecasting	126
4.8.1	<i>Prediction Intervals</i>	126
4.8.2	<i>Predicting <math>y</math> when the Regression Model Describes Log <math>y</math></i>	127
4.8.3	<i>Prediction Interval for <math>y</math> when the Regression Model Describes Log <math>y</math></i>	128
4.8.4	<i>Forecasting</i>	132
4.9	Data Problems	133
4.9.1	<i>Multicollinearity</i>	134
4.9.2	<i>Principal Components</i>	137
4.9.3	<i>Missing Values and Data Imputation</i>	138
4.9.4	<i>Measurement Error</i>	142
4.9.5	<i>Outliers and Influential Observations</i>	144
4.10	Summary and Conclusions	147

## CHAPTER 5 Hypothesis Tests and Model Selection 153

5.1	Introduction	153
5.2	Hypothesis Testing Methodology	153
5.2.1	<i>Restrictions and Hypotheses</i>	154
5.2.2	<i>Nested Models</i>	155
5.2.3	<i>Testing Procedures</i>	156
5.2.4	<i>Size, Power, and Consistency of a Test</i>	156
5.2.5	<i>A Methodological Dilemma: Bayesian Versus Classical Testing</i>	157
5.3	Three Approaches to Testing Hypotheses	157
5.3.1	<i>Wald Tests Based on the Distance Measure</i>	160
5.3.1.a	<i>Testing a Hypothesis About a Coefficient</i>	160
5.3.1.b	<i>The F Statistic</i>	163
5.3.2	<i>Tests Based on the Fit of the Regression</i>	166
5.3.2.a	<i>The Restricted Least Squares Estimator</i>	166
5.3.2.b	<i>The Loss of Fit from Restricted Least Squares</i>	167
5.3.2.c	<i>Testing the Significance of the Regression</i>	169
5.3.2.d	<i>Solving Out the Restrictions and a Caution about <math>R^2</math></i>	169
5.3.3	<i>Lagrange Multiplier Tests</i>	170
5.4	Large-Sample Tests and Robust Inference	173
5.5	Testing Nonlinear Restrictions	176
5.6	Choosing Between Nonnested Models	178
5.6.1	<i>Testing Nonnested Hypotheses</i>	179
5.6.2	<i>An Encompassing Model</i>	180
5.6.3	<i>Comprehensive Approach—The J Test</i>	180
5.7	A Specification Test	181
5.8	Model Building—A General to Simple Strategy	183
5.8.1	<i>Model Selection Criteria</i>	183
5.8.2	<i>Model Selection</i>	184

5.8.3	<i>Classical Model Selection</i>	185
5.8.4	<i>Bayesian Model Averaging</i>	185
5.9	Summary and Conclusions	187

## CHAPTER 6 Functional Form, Difference in Differences, and Structural Change 193

6.1	Introduction	193
6.2	Using Binary Variables	193
6.2.1	<i>Binary Variables in Regression</i>	193
6.2.2	<i>Several Categories</i>	197
6.2.3	<i>Modeling Individual Heterogeneity</i>	198
6.2.4	<i>Sets of Categories</i>	202
6.2.5	<i>Threshold Effects and Categorical Variables</i>	203
6.2.6	<i>Transition Tables</i>	204
6.3	Difference in Differences Regression	207
6.3.1	<i>Treatment Effects</i>	207
6.3.2	<i>Examining the Effects of Discrete Policy Changes</i>	212
6.4	Using Regression Kinks and Discontinuities to Analyze Social Policy	216
6.4.1	<i>Regression Kinked Design</i>	216
6.4.2	<i>Regression Discontinuity Design</i>	219
6.5	Nonlinearity in the Variables	223
6.5.1	<i>Functional Forms</i>	223
6.5.2	<i>Interaction Effects</i>	225
6.5.3	<i>Identifying Nonlinearity</i>	226
6.5.4	<i>Intrinsically Linear Models</i>	228
6.6	Structural Break and Parameter Variation	231
6.6.1	<i>Different Parameter Vectors</i>	231
6.6.2	<i>Robust Tests of Structural Break with Unequal Variances</i>	233
6.6.3	<i>Pooling Regressions</i>	235
6.7	Summary And Conclusions	237

## CHAPTER 7 Nonlinear, Semiparametric, and Nonparametric Regression Models 242

7.1	Introduction	242
7.2	Nonlinear Regression Models	243
7.2.1	<i>Assumptions of the Nonlinear Regression Model</i>	243
7.2.2	<i>The Nonlinear Least Squares Estimator</i>	245
7.2.3	<i>Large-Sample Properties of the Nonlinear Least Squares Estimator</i>	247
7.2.4	<i>Robust Covariance Matrix Estimation</i>	250
7.2.5	<i>Hypothesis Testing and Parametric Restrictions</i>	251

7.2.6	<i>Applications</i>	252
7.2.7	<i>Loglinear Models</i>	255
7.2.8	<i>Computing the Nonlinear Least Squares Estimator</i>	262
7.3	Median and Quantile Regression	265
7.3.1	<i>Least Absolute Deviations Estimation</i>	266
7.3.2	<i>Quantile Regression Models</i>	268
7.4	Partially Linear Regression	274
7.5	Nonparametric Regression	275
7.6	Summary and Conclusions	278

## CHAPTER 8 Endogeneity and Instrumental Variable Estimation 282

8.1	Introduction	282
8.2	Assumptions of the Extended Model	286
8.3	Instrumental Variables Estimation	288
8.3.1	<i>Least Squares</i>	288
8.3.2	<i>The Instrumental Variables Estimator</i>	289
8.3.3	<i>Estimating the Asymptotic Covariance Matrix</i>	290
8.3.4	<i>Motivating the Instrumental Variables Estimator</i>	291
8.4	Two-Stage Least Squares, Control Functions, and Limited Information Maximum Likelihood	296
8.4.1	<i>Two-Stage Least Squares</i>	297
8.4.2	<i>A Control Function Approach</i>	299
8.4.3	<i>Limited Information Maximum Likelihood</i>	301
8.5	Endogenous Dummy Variables: Estimating Treatment Effects	302
8.5.1	<i>Regression Analysis of Treatment Effects</i>	306
8.5.2	<i>Instrumental Variables</i>	307
8.5.3	<i>A Control Function Estimator</i>	309
8.5.4	<i>Propensity Score Matching</i>	310
8.6	Hypothesis Tests	314
8.6.1	<i>Testing Restrictions</i>	314
8.6.2	<i>Specification Tests</i>	315
8.6.3	<i>Testing for Endogeneity: The Hausman and Wu Specification Tests</i>	316
8.6.4	<i>A Test for Overidentification</i>	317
8.7	Weak Instruments and LIML	319
8.8	Measurement Error	321
8.8.1	<i>Least Squares Attenuation</i>	322
8.8.2	<i>Instrumental Variables Estimation</i>	324
8.8.3	<i>Proxy Variables</i>	325
8.9	Nonlinear Instrumental Variables Estimation	328
8.10	Natural Experiments and the Search for Causal Effects	331
8.11	Summary and Conclusions	334

## Part II Generalized Regression Model and Equation Systems

### CHAPTER 9 The Generalized Regression Model and Heteroscedasticity 337

9.1	Introduction	337
9.2	Robust Least Squares Estimation and Inference	338
9.3	Properties of Least Squares and Instrumental Variables	341
9.3.1	<i>Finite-Sample Properties of Least Squares</i>	341
9.3.2	<i>Asymptotic Properties of Least Squares</i>	342
9.3.3	<i>Heteroscedasticity and <math>\text{Var}[\mathbf{b} \mathbf{X}]</math></i>	344
9.3.4	<i>Instrumental Variable Estimation</i>	345
9.4	Efficient Estimation by Generalized Least Squares	346
9.4.1	<i>Generalized Least Squares (GLS)</i>	346
9.4.2	<i>Feasible Generalized Least Squares (FGLS)</i>	349
9.5	Heteroscedasticity and Weighted Least Squares	350
9.5.1	<i>Weighted Least Squares</i>	351
9.5.2	<i>Weighted Least Squares with Known <math>\Omega</math></i>	351
9.5.3	<i>Estimation When <math>\Omega</math> Contains Unknown Parameters</i>	352
9.6	Testing for Heteroscedasticity	353
9.6.1	<i>White's General Test</i>	354
9.6.2	<i>The Lagrange Multiplier Test</i>	354
9.7	Two Applications	355
9.7.1	<i>Multiplicative Heteroscedasticity</i>	355
9.7.2	<i>Groupwise Heteroscedasticity</i>	357
9.8	Summary and Conclusions	360

### CHAPTER 10 Systems of Regression Equations 366

10.1	Introduction	366
10.2	The Seemingly Unrelated Regressions Model	368
10.2.1	<i>Ordinary Least Squares And Robust Inference</i>	370
10.2.2	<i>Generalized Least Squares</i>	372
10.2.3	<i>Feasible Generalized Least Squares</i>	373
10.2.4	<i>Testing Hypotheses</i>	374
10.2.5	<i>The Pooled Model</i>	376
10.3	Systems of Demand Equations: Singular Systems	379
10.3.1	<i>Cobb-Douglas Cost Function</i>	379
10.3.2	<i>Flexible Functional Forms: The Translog Cost Function</i>	382
10.4	Simultaneous Equations Models	386
10.4.1	<i>Systems of Equations</i>	387
10.4.2	<i>A General Notation for Linear Simultaneous Equations Models</i>	390
10.4.3	<i>The Identification Problem</i>	393
10.4.4	<i>Single Equation Estimation and Inference</i>	398
10.4.5	<i>System Methods of Estimation</i>	402
10.5	Summary and Conclusions	405

**CHAPTER 11 Models for Panel Data 413**

11.1	Introduction	413
11.2	Panel Data Modeling	414
11.2.1	<i>General Modeling Framework for Analyzing Panel Data</i>	415
11.2.2	<i>Model Structures</i>	416
11.2.3	<i>Extensions</i>	417
11.2.4	<i>Balanced and Unbalanced Panels</i>	417
11.2.5	<i>Attrition and Unbalanced Panels</i>	418
11.2.6	<i>Well-Behaved Panel Data</i>	422
11.3	The Pooled Regression Model	423
11.3.1	<i>Least Squares Estimation of the Pooled Model</i>	423
11.3.2	<i>Robust Covariance Matrix Estimation and Bootstrapping</i>	424
11.3.3	<i>Clustering and Stratification</i>	426
11.3.4	<i>Robust Estimation Using Group Means</i>	428
11.3.5	<i>Estimation with First Differences</i>	429
11.3.6	<i>The Within- and Between-Groups Estimators</i>	430
11.4	The Fixed Effects Model	433
11.4.1	<i>Least Squares Estimation</i>	433
11.4.2	<i>A Robust Covariance Matrix for <math>\mathbf{b}_{LSDV}</math></i>	436
11.4.3	<i>Testing the Significance of the Group Effects</i>	437
11.4.4	<i>Fixed Time and Group Effects</i>	438
11.4.5	<i>Reinterpreting the Within Estimator: Instrumental Variables and Control Functions</i>	439
11.4.6	<i>Parameter Heterogeneity</i>	441
11.5	Random Effects	444
11.5.1	<i>Least Squares Estimation</i>	445
11.5.2	<i>Generalized Least Squares</i>	447
11.5.3	<i>Feasible Generalized Least Squares Estimation of the Random Effects Model when <math>\Sigma</math> is Unknown</i>	448
11.5.4	<i>Robust Inference and Feasible Generalized Least Squares</i>	449
11.5.5	<i>Testing for Random Effects</i>	450
11.5.6	<i>Hausman's Specification Test for the Random Effects Model</i>	454
11.5.7	<i>Extending the Unobserved Effects Model: Mundlak's Approach</i>	455
11.5.8	<i>Extending the Random and Fixed Effects Models: Chamberlain's Approach</i>	456
11.6	Nonspherical Disturbances and Robust Covariance Matrix Estimation	461
11.6.1	<i>Heteroscedasticity in the Random Effects Model</i>	461
11.6.2	<i>Autocorrelation in Panel Data Models</i>	462
11.7	Spatial Autocorrelation	462

11.8	Endogeneity	467
11.8.1	<i>Instrumental Variable Estimation</i>	467
11.8.2	<i>Hausman and Taylor's Instrumental Variables Estimator</i>	469
11.8.3	<i>Consistent Estimation of Dynamic Panel Data Models: Anderson and Hsiao's Iv Estimator</i>	473
11.8.4	<i>Efficient Estimation of Dynamic Panel Data Models: The Arellano/Bond Estimators</i>	476
11.8.5	<i>Nonstationary Data and Panel Data Models</i>	485
11.9	Nonlinear Regression with Panel Data	486
11.9.1	<i>A Robust Covariance Matrix for Nonlinear Least Squares</i>	486
11.9.2	<i>Fixed Effects in Nonlinear Regression Models</i>	487
11.9.3	<i>Random Effects</i>	489
11.10	Parameter Heterogeneity	490
11.10.1	<i>A Random Coefficients Model</i>	490
11.10.2	<i>A Hierarchical Linear Model</i>	493
11.10.3	<i>Parameter Heterogeneity and Dynamic Panel Data Models</i>	495
11.11	Summary and Conclusions	499

### Part III    Estimation Methodology

<b>CHAPTER 12    Estimation Frameworks in Econometrics</b>		<b>505</b>
12.1	Introduction	505
12.2	Parametric Estimation and Inference	507
12.2.1	<i>Classical Likelihood-Based Estimation</i>	507
12.2.2	<i>Modeling Joint Distributions with Copula Functions</i>	509
12.3	Semiparametric Estimation	512
12.3.1	<i>Gmm Estimation in Econometrics</i>	513
12.3.2	<i>Maximum Empirical Likelihood Estimation</i>	513
12.3.3	<i>Least Absolute Deviations Estimation and Quantile Regression</i>	515
12.3.4	<i>Kernel Density Methods</i>	515
12.3.5	<i>Comparing Parametric and Semiparametric Analyses</i>	516
12.4	Nonparametric Estimation	518
12.4.1	<i>Kernel Density Estimation</i>	518
12.5	Properties of Estimators	521
12.5.1	<i>Statistical Properties of Estimators</i>	521
12.5.2	<i>Extremum Estimators</i>	522
12.5.3	<i>Assumptions for Asymptotic Properties of Extremum Estimators</i>	523
12.5.4	<i>Asymptotic Properties of Estimators</i>	525
12.5.5	<i>Testing Hypotheses</i>	527
12.6	Summary and Conclusions	527

## **CHAPTER 13 Minimum Distance Estimation and the Generalized Method of Moments 528**

13.1	Introduction	528
13.2	Consistent Estimation: The Method of Moments	529
13.2.1	<i>Random Sampling and Estimating the Parameters of Distributions</i>	530
13.2.2	<i>Asymptotic Properties of the Method of Moments Estimator</i>	533
13.2.3	<i>Summary—The Method of Moments</i>	536
13.3	Minimum Distance Estimation	536
13.4	The Generalized Method of Moments (GMM) Estimator	540
13.4.1	<i>Estimation Based on Orthogonality Conditions</i>	541
13.4.2	<i>Generalizing the Method of Moments</i>	542
13.4.3	<i>Properties of the GMM Estimator</i>	546
13.5	Testing Hypotheses in the GMM Framework	550
13.5.1	<i>Testing the Validity of the Moment Restrictions</i>	550
13.5.2	<i>Gmm Wald Counterparts to the WALD, LM, and LR Tests</i>	552
13.6	Gmm Estimation of Econometric Models	553
13.6.1	<i>Single-Equation Linear Models</i>	554
13.6.2	<i>Single-Equation Nonlinear Models</i>	559
13.6.3	<i>Seemingly Unrelated Regression Equations</i>	562
13.6.4	<i>Gmm Estimation of Dynamic Panel Data Models</i>	563
13.7	Summary and Conclusions	574

## **CHAPTER 14 Maximum Likelihood Estimation 577**

14.1	Introduction	577
14.2	The Likelihood Function and Identification of the Parameters	577
14.3	Efficient Estimation: The Principle of Maximum Likelihood	579
14.4	Properties of Maximum Likelihood Estimators	581
14.4.1	<i>Regularity Conditions</i>	582
14.4.2	<i>Properties of Regular Densities</i>	583
14.4.3	<i>The Likelihood Equation</i>	584
14.4.4	<i>The Information Matrix Equality</i>	585
14.4.5	<i>Asymptotic Properties of the Maximum Likelihood Estimator</i>	585
14.4.5.a	<i>Consistency</i>	585
14.4.5.b	<i>Asymptotic Normality</i>	587
14.4.5.c	<i>Asymptotic Efficiency</i>	588
14.4.5.d	<i>Invariance</i>	588
14.4.5.e	<i>Conclusion</i>	589
14.4.6	<i>Estimating the Asymptotic Variance of the Maximum Likelihood Estimator</i>	589
14.5	Conditional Likelihoods and Econometric Models	591

14.6	Hypothesis and Specification Tests and Fit Measures	592
14.6.1	<i>The Likelihood Ratio Test</i>	594
14.6.2	<i>The Wald Test</i>	595
14.6.3	<i>The Lagrange Multiplier Test</i>	597
14.6.4	<i>An Application of the Likelihood-Based Test Procedures</i>	598
14.6.5	<i>Comparing Models and Computing Model Fit</i>	600
14.6.6	<i>Vuong's Test and the Kullback–Leibler Information Criterion</i>	602
14.7	Two-Step Maximum Likelihood Estimation	604
14.8	Pseudo-Maximum Likelihood Estimation and Robust Asymptotic Covariance Matrices	610
14.8.1	<i>A Robust Covariance Matrix Estimator for the MLE</i>	610
14.8.2	<i>Cluster Estimators</i>	613
14.9	Maximum Likelihood Estimation of Linear Regression Models	616
14.9.1	<i>Linear Regression Model with Normally Distributed Disturbances</i>	616
14.9.2	<i>Some Linear Models with Nonnormal Disturbances</i>	618
14.9.3	<i>Hypothesis Tests for Regression Models</i>	620
14.10	The Generalized Regression Model	625
14.10.1	<i>GLS With Known <math>\Omega</math></i>	625
14.10.2	<i>Iterated Feasible GLS With Estimated <math>\Omega</math></i>	626
14.10.3	<i>Multiplicative Heteroscedasticity</i>	626
14.10.4	<i>The Method of Scoring</i>	627
14.11	Nonlinear Regression Models and Quasi-Maximum Likelihood Estimation	631
14.11.1	<i>Maximum Likelihood Estimation</i>	632
14.11.2	<i>Quasi-Maximum Likelihood Estimation</i>	635
14.12	Systems of Regression Equations	640
14.12.1	<i>The Pooled Model</i>	640
14.12.2	<i>The SUR Model</i>	641
14.13	Simultaneous Equations Models	644
14.14	Panel Data Applications	645
14.14.1	<i>ML Estimation of the Linear Random Effects Model</i>	646
14.14.2	<i>Nested Random Effects</i>	649
14.14.3	<i>Clustering Over More than One Level</i>	652
14.14.4	<i>Random Effects in Nonlinear Models: MLE Using Quadrature</i>	653
14.14.5	<i>Fixed Effects in Nonlinear Models: The Incidental Parameters Problem</i>	657
14.15	Latent Class and Finite Mixture Models	662
14.15.1	<i>A Finite Mixture Model</i>	662
14.15.2	<i>Modeling the Class Probabilities</i>	664

14.15.3	<i>Latent Class Regression Models</i>	665
14.15.4	<i>Predicting Class Membership and <math>\mathbf{B}_i</math></i>	666
14.15.5	<i>Determining the Number of Classes</i>	668
14.15.6	<i>A Panel Data Application</i>	668
14.15.7	<i>A Semiparametric Random Effects Model</i>	673
14.16	Summary and Conclusions	675

## CHAPTER 15 Simulation-Based Estimation and Inference and Random Parameter Models 681

15.1	Introduction	681
15.2	Random Number Generation	683
15.2.1	<i>Generating Pseudo-Random Numbers</i>	683
15.2.2	<i>Sampling from a Standard Uniform Population</i>	684
15.2.3	<i>Sampling from Continuous Distributions</i>	685
15.2.4	<i>Sampling from a Multivariate Normal Population</i>	686
15.2.5	<i>Sampling from Discrete Populations</i>	686
15.3	Simulation-Based Statistical Inference: The Method of Krinsky and Robb	687
15.4	Bootstrapping Standard Errors and Confidence Intervals	690
15.4.1	<i>Types of Bootstraps</i>	691
15.4.2	<i>Bias Reduction with Bootstrap Estimators</i>	691
15.4.3	<i>Bootstrapping Confidence Intervals</i>	692
15.4.4	<i>Bootstrapping with Panel Data: The Block Bootstrap</i>	692
15.5	Monte Carlo Studies	693
15.5.1	<i>A Monte Carlo Study: Behavior of a Test Statistic</i>	695
15.5.2	<i>A Monte Carlo Study: The Incidental Parameters Problem</i>	696
15.6	Simulation-Based Estimation	700
15.6.1	<i>Random Effects in a Nonlinear Model</i>	701
15.6.2	<i>Monte Carlo Integration</i>	702
15.6.2a	<i>Halton Sequences and Random Draws for Simulation-Based Integration</i>	704
15.6.2.b	<i>Computing Multivariate Normal Probabilities Using the GHK Simulator</i>	706
15.6.3	<i>Simulation-Based Estimation of Random Effects Models</i>	708
15.7	A Random Parameters Linear Regression Model	713
15.8	Hierarchical Linear Models	718
15.9	Nonlinear Random Parameter Models	720
15.10	Individual Parameter Estimates	721
15.11	Mixed Models and Latent Class Models	728
15.12	Summary and Conclusions	732

<b>CHAPTER 16 Bayesian Estimation and Inference</b>	<b>734</b>
16.1 Introduction	734
16.2 Bayes' Theorem and the Posterior Density	735
16.3 Bayesian Analysis of the Classical Regression Model	737
16.3.1 <i>Analysis with a Noninformative Prior</i>	738
16.3.2 <i>Estimation with an Informative Prior Density</i>	740
16.4 Bayesian Inference	743
16.4.1 <i>Point Estimation</i>	743
16.4.2 <i>Interval Estimation</i>	744
16.4.3 <i>Hypothesis Testing</i>	745
16.4.4 <i>Large-Sample Results</i>	747
16.5 Posterior Distributions and the Gibbs Sampler	747
16.6 Application: Binomial Probit Model	750
16.7 Panel Data Application: Individual Effects Models	753
16.8 Hierarchical Bayes Estimation of a Random Parameters Model	755
16.9 Summary and Conclusions	761

## Part IV Cross Sections, Panel Data, and Microeometrics

<b>CHAPTER 17 Binary Outcomes and Discrete Choices</b>	<b>765</b>
17.1 Introduction	765
17.2 Models for Binary Outcomes	768
17.2.1 <i>Random Utility</i>	769
17.2.2 <i>The Latent Regression Model</i>	770
17.2.3 <i>Functional Form and Probability</i>	771
17.2.4 <i>Partial Effects in Binary Choice Models</i>	774
17.2.5 <i>Odds Ratios in Logit Models</i>	776
17.2.6 <i>The Linear Probability Model</i>	780
17.3 Estimation and Inference for Binary Choice Models	782
17.3.1 <i>Robust Covariance Matrix Estimation</i>	784
17.3.2 <i>Hypothesis Tests</i>	786
17.3.3 <i>Inference for Partial Effects</i>	789
17.3.3.a <i>The Delta Method</i>	789
17.3.3.b <i>An Adjustment to the Delta Method</i>	791
17.3.3.c <i>The Method of Krinsky and Robb</i>	792
17.3.3.d <i>Bootstrapping</i>	792
17.3.4 <i>Interaction Effects</i>	795
17.4 Measuring Goodness of Fit for Binary Choice Models	797
17.4.1 <i>Fit Measures Based on the Fitting Criterion</i>	797
17.4.2 <i>Fit Measures Based on Predicted Values</i>	798
17.4.3 <i>Summary of Fit Measures</i>	800
17.5 Specification Analysis	802
17.5.1 <i>Omitted Variables</i>	803

17.5.2	<i>Heteroscedasticity</i>	804
17.5.3	<i>Distributional Assumptions</i>	806
17.5.4	<i>Choice-Based Sampling</i>	808
17.6	Treatment Effects and Endogenous Variables in Binary Choice Models	809
17.6.1	<i>Endogenous Treatment Effect</i>	810
17.6.2	<i>Endogenous Continuous Variable</i>	813
17.6.2.a	<i>IV and GMM Estimation</i>	813
17.6.2.b	<i>Partial ML Estimation</i>	814
17.6.2.c	<i>Full Information Maximum Likelihood Estimation</i>	814
17.6.2.d	<i>Residual Inclusion and Control Functions</i>	815
17.6.2.e	<i>A Control Function Estimator</i>	815
17.6.3	<i>Endogenous Sampling</i>	817
17.7	Panel Data Models	820
17.7.1	<i>The Pooled Estimator</i>	821
17.7.2	<i>Random Effects</i>	822
17.7.3	<i>Fixed Effects</i>	825
17.7.3.a	<i>A Conditional Fixed Effects Estimator</i>	827
17.7.3.b	<i>Mundlak's Approach, Variable Addition, and Bias Reduction</i>	832
17.7.4	<i>Dynamic Binary Choice Models</i>	834
17.7.5	<i>A Semiparametric Model for Individual Heterogeneity</i>	837
17.7.6	<i>Modeling Parameter Heterogeneity</i>	838
17.7.7	<i>Nonresponse, Attrition, and Inverse Probability Weighting</i>	841
17.8	Spatial Binary Choice Models	844
17.9	The Bivariate Probit Model	847
17.9.1	<i>Maximum Likelihood Estimation</i>	848
17.9.2	<i>Testing for Zero Correlation</i>	851
17.9.3	<i>Partial Effects</i>	851
17.9.4	<i>A Panel Data Model for Bivariate Binary Response</i>	854
17.9.5	<i>A Recursive Bivariate Probit Model</i>	855
17.10	A Multivariate Probit Model	859
17.11	Summary and Conclusions	862

## CHAPTER 18 Multinomial Choices and Event Counts 866

18.1	Introduction	866
18.2	Models for Unordered Multiple Choices	867
18.2.1	<i>Random Utility Basis of the Multinomial Logit Model</i>	867
18.2.2	<i>The Multinomial Logit Model</i>	869
18.2.3	<i>The Conditional Logit Model</i>	873
18.2.4	<i>The Independence from Irrelevant Alternatives Assumption</i>	874
18.2.5	<i>Alternative Choice Models</i>	875
18.2.5.a	<i>Heteroscedastic Extreme Value Model</i>	876

	<i>18.2.5.b Multinomial Probit Model</i>	876
	<i>18.2.5.c The Nested Logit Model</i>	877
18.2.6	<i>Modeling Heterogeneity</i>	885
	<i>18.2.6.a The Mixed Logit Model</i>	885
	<i>18.2.6.b A Generalized Mixed Logit Model</i>	886
	<i>18.2.6.c Latent Classes</i>	889
	<i>18.2.6.d Attribute Nonattendance</i>	891
18.2.7	<i>Estimating Willingness to Pay</i>	893
18.2.8	<i>Panel Data and Stated Choice Experiments</i>	896
	<i>18.2.8.a The Mixed Logit Model</i>	897
	<i>18.2.8.b Random Effects and the Nested Logit Model</i>	898
	<i>18.2.8.c A Fixed Effects Multinomial Logit Model</i>	899
18.2.9	<i>Aggregate Market Share Data—The BLP Random Parameters Model</i>	903
18.3	<b>Random Utility Models for Ordered Choices</b>	905
18.3.1	<i>The Ordered Probit Model</i>	909
18.3.2.A	<i>Specification Test for the Ordered Choice Model</i>	912
18.3.3	<i>Bivariate Ordered Probit Models</i>	913
18.3.4	<i>Panel Data Applications</i>	915
	<i>18.3.4.a Ordered Probit Models with Fixed Effects</i>	915
	<i>18.3.4.b Ordered Probit Models with Random Effects</i>	917
18.3.5	<i>Extensions of the Ordered Probit Model</i>	921
	<i>18.3.5.a Threshold Models—Generalized Ordered Choice Models</i>	921
	<i>18.3.5.b Thresholds and Heterogeneity—Anchoring Vignettes</i>	923
18.4	<b>Models for Counts of Events</b>	924
18.4.1	<i>The Poisson Regression Model</i>	925
18.4.2	<i>Measuring Goodness of Fit</i>	927
18.4.3	<i>Testing for Overdispersion</i>	928
18.4.4	<i>Heterogeneity and the Negative Binomial Regression Model</i>	929
18.4.5	<i>Functional Forms for Count Data Models</i>	930
18.4.6	<i>Truncation and Censoring in Models for Counts</i>	934
18.4.7	<i>Panel Data Models</i>	938
	<i>18.4.7.a Robust Covariance Matrices for Pooled Estimators</i>	938
	<i>18.4.7.b Fixed Effects</i>	940
	<i>18.4.7.c Random Effects</i>	942
18.4.8	<i>Two-Part Models: Zero-Inflation and Hurdle Models</i>	945
18.4.9	<i>Endogenous Variables and Endogenous Participation</i>	950
18.5	<b>Summary and Conclusions</b>	954

## CHAPTER 19 Limited Dependent Variables—Truncation, Censoring, and Sample Selection    958

19.1	<b>Introduction</b>	958
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19.2	Truncation	958
19.2.1	<i>Truncated Distributions</i>	959
19.2.2	<i>Moments of Truncated Distributions</i>	960
19.2.3	<i>The Truncated Regression Model</i>	962
19.2.4	<i>The Stochastic Frontier Model</i>	964
19.3	Censored Data	970
19.3.1	<i>The Censored Normal Distribution</i>	971
19.3.2	<i>The Censored Regression (Tobit) Model</i>	973
19.3.3	<i>Estimation</i>	976
19.3.4	<i>Two-Part Models and Corner Solutions</i>	978
19.3.5	<i>Specification Issues</i>	984
19.3.5.a	<i>Endogenous Right-Hand-Side Variables</i>	984
19.3.5.b	<i>Heteroscedasticity</i>	985
19.3.5.c	<i>Nonnormality</i>	987
19.3.6	<i>Panel Data Applications</i>	988
19.4	Sample Selection and Incidental Truncation	989
19.4.1	<i>Incidental Truncation in a Bivariate Distribution</i>	989
19.4.2	<i>Regression in a Model of Selection</i>	990
19.4.3	<i>Two-Step and Maximum Likelihood Estimation</i>	993
19.4.4	<i>Sample Selection in Nonlinear Models</i>	997
19.4.5	<i>Panel Data Applications of Sample Selection Models</i>	1001
19.4.5.a	<i>Common Effects in Sample Selection Models</i>	1001
19.4.5.b	<i>Attrition</i>	1004
19.5	Models for Duration	1005
19.5.1	<i>Models for Duration Data</i>	1006
19.5.2	<i>Duration Data</i>	1006
19.5.3	<i>A Regression-Like Approach: Parametric Models of Duration</i>	1007
19.5.3.a	<i>Theoretical Background</i>	1007
19.5.3.b	<i>Models of the Hazard Function</i>	1008
19.5.3.c	<i>Maximum Likelihood Estimation</i>	1010
19.5.3.d	<i>Exogenous Variables</i>	1011
19.5.3.e	<i>Heterogeneity</i>	1012
19.5.4	<i>Nonparametric and Semiparametric Approaches</i>	1013
19.6	Summary and Conclusions	1016

## Part V Time Series and Macroeconometrics

### CHAPTER 20 Serial Correlation 1021

20.1	Introduction	1021
20.2	The Analysis of Time-Series Data	1024
20.3	Disturbance Processes	1027
20.3.1	<i>Characteristics of Disturbance Processes</i>	1027
20.3.2	<i>Ar(1) Disturbances</i>	1029
20.4	Some Asymptotic Results for Analyzing Time-Series Data	1030

20.4.1	<i>Convergence of Moments—The Ergodic Theorem</i>	1031
20.4.2	<i>Convergence to Normality—A Central Limit Theorem</i>	1034
20.5	Least Squares Estimation	1036
20.5.1	<i>Asymptotic Properties of Least Squares</i>	1036
20.5.2	<i>Estimating the Variance of the Least Squares Estimator</i>	1038
20.6	Gmm Estimation	1039
20.7	Testing for Autocorrelation	1040
20.7.1	<i>Lagrange Multiplier Test</i>	1040
20.7.2	<i>Box And Pierce's Test and Ljung's Refinement</i>	1041
20.7.3	<i>The Durbin–Watson Test</i>	1041
20.7.4	<i>Testing in the Presence of a Lagged Dependent Variable</i>	1042
20.7.5	<i>Summary of Testing Procedures</i>	1042
20.8	Efficient Estimation when $\Omega$ is Known	1043
20.9	Estimation when $\Omega$ Unknown	1044
20.9.1	<i>Ar(1) Disturbances</i>	1044
20.9.2	<i>Application: Estimation of a Model with Autocorrelation</i>	1045
20.9.3	<i>Estimation with a Lagged Dependent Variable</i>	1047
20.10	Autoregressive Conditional Heteroscedasticity	1050
20.10.1	<i>The ARCH(1) Model</i>	1051
20.10.2	<i>ARCH(<math>q</math>), ARCH-In-Mean, and Generalized ARCH Models</i>	1052
20.10.3	<i>Maximum Likelihood Estimation of the GARCH Model</i>	1054
20.10.4	<i>Testing for GARCH Effects</i>	1057
20.10.5	<i>Pseudo–Maximum Likelihood Estimation</i>	1058
20.11	Summary and Conclusions	1059

**CHAPTER 21 Nonstationary Data    1062**

21.1	Introduction	1062
21.2	Nonstationary Processes and Unit Roots	1062
21.2.1	<i>The Lag and Difference Operators</i>	1062
21.2.2	<i>Integrated Processes and Differencing</i>	1063
21.2.3	<i>Random Walks, Trends, and Spurious Regressions</i>	1066
21.2.4	<i>Tests for Unit Roots in Economic Data</i>	1068
21.2.5	<i>The Dickey–Fuller Tests</i>	1069
21.2.6	<i>The KPSS Test of Stationarity</i>	1078
21.3	Cointegration	1079
21.3.1	<i>Common Trends</i>	1083
21.3.2	<i>Error Correction and Var Representations</i>	1084
21.3.3	<i>Testing for Cointegration</i>	1085
21.3.4	<i>Estimating Cointegration Relationships</i>	1088
21.3.5	<i>Application: German Money Demand</i>	1088
21.3.5.a	<i>Cointegration Analysis and a Long-Run Theoretical Model</i>	1089

21.3.5.b <i>Testing for Model Instability</i>	<i>1090</i>
21.4 Nonstationary Panel Data	1091
21.5 Summary and Conclusions	1092
<b>References</b>	<b>1094</b>
<b>Index</b>	<b>1138</b>
 <b>Part VI Online Appendices</b>	
<b>Appendix A Matrix Algebra</b>	<b>A-1</b>
A.1 Terminology	A-1
A.2 Algebraic Manipulation of Matrices	A-2
A.2.1 <i>Equality of Matrices</i>	A-2
A.2.2 <i>Transposition</i>	A-2
A.2.3 <i>Vectorization</i>	A-3
A.2.4 <i>Matrix Addition</i>	A-3
A.2.5 <i>Vector Multiplication</i>	A-3
A.2.6 <i>A Notation for Rows and Columns of a Matrix</i>	A-3
A.2.7 <i>Matrix Multiplication and Scalar Multiplication</i>	A-4
A.2.8 <i>Sums of Values</i>	A-5
A.2.9 <i>A Useful Idempotent Matrix</i>	A-6
A.3 Geometry of Matrices	A-8
A.3.1 <i>Vector Spaces</i>	A-8
A.3.2 <i>Linear Combinations of Vectors and Basis Vectors</i>	A-9
A.3.3 <i>Linear Dependence</i>	A-11
A.3.4 <i>Subspaces</i>	A-12
A.3.5 <i>Rank of a Matrix</i>	A-12
A.3.6 <i>Determinant of a Matrix</i>	A-15
A.3.7 <i>A Least Squares Problem</i>	A-16
A.4 Solution of a System of Linear Equations	A-19
A.4.1 <i>Systems of Linear Equations</i>	A-19
A.4.2 <i>Inverse Matrices</i>	A-19
A.4.3 <i>Nonhomogeneous Systems of Equations</i>	A-21
A.4.4 <i>Solving the Least Squares Problem</i>	A-21
A.5 Partitioned Matrices	A-22
A.5.1 <i>Addition and Multiplication of Partitioned Matrices</i>	A-22
A.5.2 <i>Determinants of Partitioned Matrices</i>	A-23
A.5.3 <i>Inverses of Partitioned Matrices</i>	A-23
A.5.4 <i>Deviations From Means</i>	A-23
A.5.5 <i>Kronecker Products</i>	A-24
A.6 Characteristic Roots And Vectors	A-24
A.6.1 <i>The Characteristic Equation</i>	A-25
A.6.2 <i>Characteristic Vectors</i>	A-25
A.6.3 <i>General Results for Characteristic Roots And Vectors</i>	A-26

A.6.4	<i>Diagonalization and Spectral Decomposition of a Matrix</i>	A-26
A.6.5	<i>Rank of a Matrix</i>	A-27
A.6.6	<i>Condition Number of a Matrix</i>	A-28
A.6.7	<i>Trace of a Matrix</i>	A-29
A.6.8	<i>Determinant of a Matrix</i>	A-30
A.6.9	<i>Powers of a Matrix</i>	A-30
A.6.10	<i>Idempotent Matrices</i>	A-32
A.6.11	<i>Factoring a Matrix: The Cholesky Decomposition</i>	A-32
A.6.12	<i>Singular Value Decomposition</i>	A-33
A.6.13	<i>Qr Decomposition</i>	A-33
A.6.14	<i>The Generalized Inverse of a Matrix</i>	A-33
A.7	<b>Quadratic Forms And Definite Matrices</b>	A-34
A.7.1	<i>Nonnegative Definite Matrices</i>	A-35
A.7.2	<i>Idempotent Quadratic Forms</i>	A-36
A.7.3	<i>Comparing Matrices</i>	A-37
A.8	<b>Calculus And Matrix Algebra</b>	A-37
A.8.1	<i>Differentiation and the Taylor Series</i>	A-37
A.8.2	<i>Optimization</i>	A-41
A.8.3	<i>Constrained Optimization</i>	A-43
A.8.4	<i>Transformations</i>	A-45

## Appendix B    Probability and Distribution

### Theory    B-1

B.1	<b>Introduction</b>	B-1
B.2	<b>Random Variables</b>	B-1
B.2.1	<i>Probability Distributions</i>	B-2
B.2.2	<i>Cumulative Distribution Function</i>	B-2
B.3	<b>Expectations of a Random Variable</b>	B-3
B.4	<b>Some Specific Probability Distributions</b>	B-6
B.4.1	<i>The Normal and Skew Normal Distributions</i>	B-6
B.4.2	<i>The Chi-Squared, t, and F Distributions</i>	B-8
B.4.3	<i>Distributions with Large Degrees of Freedom</i>	B-11
B.4.4	<i>Size Distributions: The Lognormal Distribution</i>	B-12
B.4.5	<i>The Gamma and Exponential Distributions</i>	B-13
B.4.6	<i>The Beta Distribution</i>	B-13
B.4.7	<i>The Logistic Distribution</i>	B-14
B.4.8	<i>The Wishart Distribution</i>	B-14
B.4.9	<i>Discrete Random Variables</i>	B-15
B.5	<b>The Distribution of a Function of a Random Variable</b>	B-15
B.6	<b>Representations of a Probability Distribution</b>	B-18
B.7	<b>Joint Distributions</b>	B-19
B.7.1	<i>Marginal Distributions</i>	B-20
B.7.2	<i>Expectations in a Joint Distribution</i>	B-20
B.7.3	<i>Covariance and Correlation</i>	B-21

B.7.4	<i>Distribution of a Function of Bivariate Random Variables</i>	B-22
B.8	Conditioning in a Bivariate Distribution	B-23
B.8.1	<i>Regression: The Conditional Mean</i>	B-24
B.8.2	<i>Conditional Variance</i>	B-24
B.8.3	<i>Relationships among Marginal and Conditional Moments</i>	B-24
B.8.4	<i>The Analysis of Variance</i>	B-26
B.8.5	<i>Linear Projection</i>	B-27
B.9	The Bivariate Normal Distribution	B-28
B.10	Multivariate Distributions	B-29
B.10.1	<i>Moments</i>	B-29
B.10.2	<i>Sets of Linear Functions</i>	B-30
B.10.3	<i>Nonlinear Functions: The Delta Method</i>	B-31
B.11	The Multivariate Normal Distribution	B-31
B.11.1	<i>Marginal and Conditional Normal Distributions</i>	B-32
B.11.2	<i>The Classical Normal Linear Regression Model</i>	B-33
B.11.3	<i>Linear Functions of a Normal Vector</i>	B-33
B.11.4	<i>Quadratic Forms in a Standard Normal Vector</i>	B-34
B.11.5	<i>The F Distribution</i>	B-36
B.11.6	<i>A Full Rank Quadratic Form</i>	B-36
B.11.7	<i>Independence of a Linear and a Quadratic Form</i>	B-38

## Appendix C Estimation and Inference C-1

C.1	Introduction	C-1
C.2	Samples and Random Sampling	C-1
C.3	Descriptive Statistics	C-2
C.4	Statistics as Estimators—Sampling Distributions	C-6
C.5	Point Estimation of Parameters	C-9
C.5.1	<i>Estimation in a Finite Sample</i>	C-9
C.5.2	<i>Efficient Unbiased Estimation</i>	C-12
C.6	Interval Estimation	C-14
C.7	Hypothesis Testing	C-16
C.7.1	<i>Classical Testing Procedures</i>	C-16
C.7.2	<i>Tests Based on Confidence Intervals</i>	C-19
C.7.3	<i>Specification Tests</i>	D-1

## Appendix D Large-Sample Distribution Theory D-1

D.1	Introduction	D-1
D.2	Large-Sample Distribution Theory	D-2
D.2.1	<i>Convergence in Probability</i>	D-2
D.2.2	<i>Other forms of Convergence and Laws of Large Numbers</i>	D-5
D.2.3	<i>Convergence of Functions</i>	D-9
D.2.4	<i>Convergence to a Random Variable</i>	D-10

	<i>D.2.5</i>	<i>Convergence in Distribution: Limiting Distributions</i>	<i>D-11</i>
	<i>D.2.6</i>	<i>Central Limit Theorems</i>	<i>D-14</i>
	<i>D.2.7</i>	<i>The Delta Method</i>	<i>D-19</i>
<b>D.3</b>	<b>Asymptotic Distributions</b>	<b>D-19</b>	
	<i>D.3.1</i>	<i>Asymptotic Distribution of a Nonlinear Function</i>	<i>D-21</i>
	<i>D.3.2</i>	<i>Asymptotic Expectations</i>	<i>D-22</i>
<b>D.4</b>	<b>Sequences and the Order of a Sequence</b>	<b>D-24</b>	

**Appendix E   Computation and Optimization   E-1**

<b>E.1</b>	<b>Introduction</b>	<b>E-1</b>	
<b>E.2</b>	<b>Computation in Econometrics</b>	<b>E-1</b>	
	<i>E.2.1</i>	<i>Computing Integrals</i>	<i>E-2</i>
	<i>E.2.2</i>	<i>The Standard Normal Cumulative Distribution Function</i>	<i>E-2</i>
	<i>E.2.3</i>	<i>The Gamma and Related Functions</i>	<i>E-3</i>
	<i>E.2.4</i>	<i>Approximating Integrals by Quadrature</i>	<i>E-4</i>
<b>E.3</b>	<b>Optimization</b>	<b>E-5</b>	
	<i>E.3.1</i>	<i>Algorithms</i>	<i>E-7</i>
	<i>E.3.2</i>	<i>Computing Derivatives</i>	<i>E-7</i>
	<i>E.3.3</i>	<i>Gradient Methods</i>	<i>E-9</i>
	<i>E.3.4</i>	<i>Aspects of Maximum Likelihood Estimation</i>	<i>E-12</i>
	<i>E.3.5</i>	<i>Optimization with Constraints</i>	<i>E-14</i>
	<i>E.3.6</i>	<i>Some Practical Considerations</i>	<i>E-15</i>
	<i>E.3.7</i>	<i>The EM Algorithm</i>	<i>E-17</i>
<b>E.4</b>	<b>Examples</b>	<b>E-19</b>	
	<i>E.4.1</i>	<i>Function of one Parameter</i>	<i>E-19</i>
	<i>E.4.2</i>	<i>Function of two Parameters: The Gamma Distribution</i>	<i>E-20</i>
	<i>E.4.3</i>	<i>A Concentrated Log-Likelihood Function</i>	<i>E-21</i>

**Appendix F   Data Sets Used in Applications   F-1**

## **EXAMPLES AND APPLICATIONS**



### **CHAPTER 1 Econometrics 41**

Example 1.1	Behavioral Models and the Nobel Laureates	42
Example 1.2	Keynes's Consumption Function	45

### **CHAPTER 2 The Linear Regression Model 52**

Example 2.1	Keynes's Consumption Function	54
Example 2.2	Earnings and Education	55
Example 2.3	The U.S. Gasoline Market	59
Example 2.4	The Translog Model	59
Example 2.5	Short Rank	60
Example 2.6	An Inestimable Model	61
Example 2.7	Nonzero Conditional Mean of the Disturbances	62

### **CHAPTER 3 Least Squares Regression 68**

Example 3.1	Partial Correlations	81
Example 3.2	Fit of a Consumption Function	84
Example 3.3	Analysis of Variance for the Investment Equation	84
Example 3.4	Art Appreciation	88

### **CHAPTER 4 Estimating the Regression Model by Least Squares 94**

Example 4.1	The Sampling Distribution of a Least Squares Estimator	98
Example 4.2	Omitted Variable in a Demand Equation	99
Example 4.3	Least Squares Vs. Least Absolute Deviations—A Monte Carlo Study	108
Example 4.4	Linear Projection: A Sampling Experiment	112
Example 4.5	Robust Inference about the Art Market	116
Example 4.6	Clustering and Block Bootstrapping	118
Example 4.7	Nonlinear Functions of Parameters: The Delta Method	120
Example 4.8	Confidence Interval for the Income Elasticity of Demand for Gasoline	123
Example 4.9	Oaxaca Decomposition of Home Sale Prices	125
Example 4.10	Pricing Art	130
Example 4.11	Multicollinearity in the Longley Data	135
Example 4.12	Predicting Movie Success	137
Example 4.13	Imputation in the Survey of Consumer Finances	141

**CHAPTER 5 Hypothesis Tests and Model Selection 153**

Example 5.1	Art Appreciation	161
Example 5.2	Earnings Equation	162
Example 5.3	Restricted Investment Equation	164
Example 5.4	F Test for the Earnings Equation	169
Example 5.5	Production Functions	170
Example 5.6	A Long-Run Marginal Propensity to Consume	177
Example 5.7	J Test for a Consumption Function	181
Example 5.8	Size of a RESET Test	182
Example 5.9	Bayesian Averaging of Classical Estimates	187

**CHAPTER 6 Functional Form, Difference in Differences, and Structural Change 193**

Example 6.1	Dummy Variable in an Earnings Equation	194
Example 6.2	Value of a Signature	195
Example 6.3	Gender and Time Effects in a Log Wage Equation	196
Example 6.4	Genre Effects on Movie Box Office Receipts	198
Example 6.5	Sports Economics: Using Dummy Variables for Unobserved Heterogeneity	200
Example 6.6	Analysis of Covariance	202
Example 6.7	Education Thresholds in a Log Wage Equation	205
Example 6.8	SAT Scores	209
Example 6.9	A Natural Experiment: The Mariel Boatlift	209
Example 6.10	Effect of the Minimum Wage	210
Example 6.11	Difference in Differences Analysis of a Price Fixing Conspiracy	212
Example 6.12	Policy Analysis Using Kinked Regressions	218
Example 6.13	The Treatment Effect of Compulsory Schooling	220
Example 6.14	Interest Elasticity of Mortgage Demand	220
Example 6.15	Quadratic Regression	224
Example 6.16	Partial Effects in a Model with Interactions	226
Example 6.17	Functional Form for a Nonlinear Cost Function	227
Example 6.18	Intrinsically Linear Regression	229
Example 6.19	CES Production Function	230
Example 6.20	Structural Break in the Gasoline Market	232
Example 6.21	Sample Partitioning by Gender	234
Example 6.22	The World Health Report	234
Example 6.23	Pooling in a Log Wage Model	236

**CHAPTER 7 Nonlinear, Semiparametric, and Nonparametric Regression Models 242**

Example 7.1	CES Production Function	243
Example 7.2	Identification in a Translog Demand System	244
Example 7.3	First-Order Conditions for a Nonlinear Model	246
Example 7.4	Analysis of a Nonlinear Consumption Function	253

Example 7.5	The Box–Cox Transformation	254
Example 7.6	Interaction Effects in a Loglinear Model for Income	256
Example 7.7	Generalized Linear Models for the Distribution of Healthcare Costs	261
Example 7.8	Linearized Regression	263
Example 7.9	Nonlinear Least Squares	264
Example 7.10	LAD Estimation of a Cobb–Douglas Production Function	268
Example 7.11	Quantile Regression for Smoking Behavior	270
Example 7.12	Income Elasticity of Credit Card Expenditures	271
Example 7.13	Partially Linear Translog Cost Function	275
Example 7.14	A Nonparametric Average Cost Function	277
<b>CHAPTER 8 Endogeneity and Instrumental Variable Estimation</b>		<b>282</b>
Example 8.1	Models with Endogenous Right-Hand-Side Variables	282
Example 8.2	Instrumental Variable Analysis	292
Example 8.3	Streams as Instruments	294
Example 8.4	Instrumental Variable in Regression	295
Example 8.5	Instrumental Variable Estimation of a Labor Supply Equation	298
Example 8.6	German Labor Market Interventions	305
Example 8.7	Treatment Effects on Earnings	306
Example 8.8	The Oregon Health Insurance Experiment	306
Example 8.9	The Effect of Counseling on Financial Management	306
Example 8.10	Treatment Effects on Earnings	311
Example 8.5	Labor Supply Model (Continued)	317
Example 8.11	Overidentification of the Labor Supply Equation	319
Example 8.12	Income and Education in a Study of Twins	326
Example 8.13	Instrumental Variables Estimates of the Consumption Function	331
Example 8.14	Does Television Watching Cause Autism?	332
Example 8.15	Is Season of Birth a Valid Instrument?	334
<b>CHAPTER 9 The Generalized Regression Model and Heteroscedasticity</b>		<b>337</b>
Example 9.1	Heteroscedastic Regression and the White Estimator	340
Example 9.2	Testing for Heteroscedasticity	355
Example 9.3	Multiplicative Heteroscedasticity	355
Example 9.4	Groupwise Heteroscedasticity	358
<b>CHAPTER 10 Systems of Regression Equations</b>		<b>366</b>
Example 10.1	A Regional Production Model for Public Capital	376
Example 10.2	Cobb–Douglas Cost Function	380
Example 10.3	A Cost Function for U.S. Manufacturing	384
Example 10.4	Reverse Causality and Endogeneity in Health	387

Example 10.5	Structure and Reduced Form in a Small Macroeconomic Model	391
Example 10.6	Identification of a Supply and Demand Model	395
Example 10.7	The Rank Condition and a Two-Equation Model	397
Example 10.8	Simultaneity in Health Production	400
Example 10.9	Klein's Model I	404

**CHAPTER 11 Models for Panel Data 413**

Example 11.1	A Rotating Panel: The Survey of Income and Program Participation (SIPP) Data	418
Example 11.2	Attrition and Inverse Probability Weighting in a Model for Health	418
Example 11.3	Attrition and Sample Selection in an Earnings Model for Physicians	420
Example 11.4	Wage Equation	425
Example 11.5	Robust Estimators of the Wage Equation	429
Example 11.6	Analysis of Covariance and the World Health Organization (WHO) Data	432
Example 11.7	Fixed Effects Estimates of a Wage Equation	437
Example 11.8	Two-Way Fixed Effects with Unbalanced Panel Data	439
Example 11.9	Heterogeneity in Time Trends in an Aggregate Production Function	442
Example 11.10	Test for Random Effects	451
Example 11.11	Estimates of the Random Effects Model	452
Example 11.12	Hausman and Variable Addition Tests for Fixed versus Random Effects	456
Example 11.13	Hospital Costs	459
Example 11.14	Spatial Autocorrelation in Real Estate Sales	464
Example 11.15	Spatial Lags in Health Expenditures	466
Example 11.16	Endogenous Income in a Health Production Model	469
Example 11.17	The Returns to Schooling	472
Example 11.18	The Returns to Schooling	473
Example 11.19	Dynamic Labor Supply Equation	483
Example 11.20	Health Care Utilization	486
Example 11.21	Exponential Model with Fixed Effects	488
Example 11.22	Random Coefficients Model	492
Example 11.23	Fannie Mae's Pass Through	493
Example 11.24	Dynamic Panel Data Models	495
Example 11.25	A Mixed Fixed Growth Model for Developing Countries	499

**CHAPTER 12 Estimation Frameworks in Econometrics 505**

Example 12.1	The Linear Regression Model	508
Example 12.2	The Stochastic Frontier Model	508
Example 12.3	Joint Modeling of a Pair of Event Counts	512

Example 12.4	The Formula That Killed Wall Street	512
Example 12.5	Semiparametric Estimator for Binary Choice Models	515
Example 12.6	A Model of Vacation Expenditures	516

## **CHAPTER 13 Minimum Distance Estimation and the Generalized Method of Moments **528****

Example 13.1	Euler Equations and Life Cycle Consumption	528
Example 13.2	Method of Moments Estimator for $N[\mu, \sigma^2]$	530
Example 13.3	Inverse Gaussian (Wald) Distribution	531
Example 13.4	Mixture of Normal Distributions	531
Example 13.5	Gamma Distribution	533
Example 13.5	(Continued)	535
Example 13.6	Minimum Distance Estimation of a Hospital Cost Function	538
Example 13.7	GMM Estimation of a Nonlinear Regression Model	544
Example 13.8	Empirical Moment Equation for Instrumental Variables	547
Example 13.9	Overidentifying Restrictions	551
Example 13.10	GMM Estimation of a Dynamic Panel Data Model of Local Government Expenditures	570

## **CHAPTER 14 Maximum Likelihood Estimation **577****

Example 14.1	Identification of Parameters	578
Example 14.2	Log-Likelihood Function and Likelihood Equations for the Normal Distribution	581
Example 14.3	Information Matrix for the Normal Distribution	588
Example 14.4	Variance Estimators for an MLE	590
Example 14.5	Two-Step ML Estimation	607
Example 14.6	A Regression with Nonnormal Disturbances	612
Example 14.7	Cluster Robust Standard Errors	614
Example 14.8	Logistic, t, and Skew Normal Disturbances	619
Example 14.9	Testing for Constant Returns to Scale	624
Example 14.10	Multiplicative Heteroscedasticity	629
Example 14.11	Maximum Likelihood Estimation of Gasoline Demand	630
Example 14.12	Identification in a Loglinear Regression Model	631
Example 14.13	Geometric Regression Model for Doctor Visits	637
Example 14.14	MI Estimates of a Seemingly Unrelated Regressions Model	642
Example 14.15	Maximum Likelihood and FGLS Estimates of a Wage Equation	648
Example 14.16	Statewide Productivity	650
Example 14.17	Random Effects Geometric Regression Model	657
Example 14.18	Fixed and Random Effects Geometric Regression	661
Example 14.19	A Normal Mixture Model for Grade Point Averages	663

Example 14.20	Latent Class Regression Model for Grade Point Averages	665
Example 14.21	Predicting Class Probabilities	667
Example 14.22	A Latent Class Two-Part Model for Health Care Utilization	670
Example 14.23	Latent Class Models for Health Care Utilization	671
Example 14.24	Semiparametric Random Effects Model	674

## **CHAPTER 15 Simulation-Based Estimation and Inference and Random Parameter Models 681**

Example 15.1	Inferring the Sampling Distribution of the Least Squares Estimator	681
Example 15.2	Bootstrapping the Variance of the LAD Estimator	681
Example 15.3	Least Simulated Sum of Squares	682
Example 15.4	Long-Run Elasticities	688
Example 15.5	Bootstrapping the Variance of the Median	691
Example 15.6	Block Bootstrapping Standard Errors and Confidence Intervals in a Panel	693
Example 15.7	Monte Carlo Study of the Mean Versus the Median	694
Example 15.8	Fractional Moments of the Truncated Normal Distribution	703
Example 15.9	Estimating the Lognormal Mean	706
Example 15.10	Poisson Regression Model with Random Effects	712
Example 15.11	Maximum Simulated Likelihood Estimation of the Random Effects Linear Regression Model	712
Example 15.12	Random Parameters Wage Equation	715
Example 15.13	Least Simulated Sum of Squares Estimates of a Production Function Model	717
Example 15.14	Hierarchical Linear Model of Home Prices	719
Example 15.15	Individual State Estimates of a Private Capital Coefficient	724
Example 15.16	Mixed Linear Model for Wages	725
Example 15.17	Maximum Simulated Likelihood Estimation of a Binary Choice Model	729

## **CHAPTER 16 Bayesian Estimation and Inference 734**

Example 16.1	Bayesian Estimation of a Probability	736
Example 16.2	Estimation with a Conjugate Prior	741
Example 16.3	Bayesian Estimate of the Marginal Propensity to Consume	743
Example 16.4	Posterior Odds for the Classical Regression Model	746
Example 16.5	Gibbs Sampling from the Normal Distribution	748
Example 16.6	Gibbs Sampler for a Probit Model	752
Example 16.7	Bayesian and Classical Estimation of Heterogeneity in the Returns to Education	757